GREAT-CIRCLE SAILING

MADE EASY;

OR THE

METHOD OF CALCULATING WITH ACCURACY & EASE

THE

SEVERAL PARTS REQUIRED FOR THE PRACTICE OF SAILING

APPROXIMATELY TO A GREAT CIRCLE.

BY JOHN SEATON,
TEACHER OF NAVIGATION AND THE MATHEMATICS,
45, PRINCE STREET, BRISTOL.

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ADDRESS.

As the subject of Great-Circle Sailing has of late occupied so large a share of public attention, no apology is made for this humble attempt to simplify and improve the application of an art so conducive to the advantage of those who may adopt it. The plan of the work was formed before the writer had any knowledge of the existence of those useful Tables, calculated by Mr. Towson, and having obtained them, he saw no reason to withhold its publication, but it rather encouraged him to proceed. He conceives that these two works will not be rivals but assistants, and he who possesses one should procure the other also. The accuracy, which, the application of the rules now given in this work will afford, seems necessary to ensure confidence; and should this effort to promote a public good, obtain a share of public approbation the labours of the writer will obtain an ample reward.

GREAT-CIRCLE SAILING.

INTRODUCTION.

1.—THE following pages have been penned with the design of laying before the Mercantile Public, the practicability, utility, and simplicity of a portion of the art of Navigation, which has hitherto been deemed as unimportant, and if not impracticable, at least difficult of execution. It is however hoped that the following pages will assist in dispelling this delusion. The practice of Great-Circle Sailing is both important and easy. It is not more difficult either to comprehend or execute than several problems which are deemed indispensable in common *Navigation*. The extra labour it imposes on the mariner is trifling in comparison with the advantages it bestows. It is also some recommendation that this little labour can be performed at home before the voyage commences. The requisite calculations, unless the voyage is a very complex one, may generally be performed in less than an hour; and they may even be done by proxy. Nor is this all, the calculations will serve for the return voyage, and for any number of voyages to and from the same ports.

2.—The art of Great-Circle Sailing has of late received an impetus, from the fact of an enterprising individual having, by adopting these principles, performed the voyage to Australia in the shortest time of any on record. The thanks of every man that is in any way interested in the maritime prosperity of our country, is due to Captain Godfrey for making the interesting experiment. It is not every man possessing the intelligence to perceive the advantages that would accrue to the public, if the sailing on or near a Great Circle could be proved practicable, that would have sufficient firmness to try the experiment at his own risk. Captain Godfrey did this, and was successful; but had he been otherwise, he must have been aware of the obloquy that would have been attached to the attempt. Had he survived, he would have met with those who would have pointed at him with the finger of scorn, with others, that would, as in the case of the unfortunate "Richard Dart," have condemned his purpose, though the wreck of this vessel

must be ascribed to an *error* either of the ship's reckoning, or of the position of the island on which she struck; which of these two was the fact, must be determined by future observations. Captain Godfrey's genius and skill overcame the difficulties, great or small, that he had to encounter; and his name, like that of Columbus, will be handed down to posterity, if not as the discoverer of new worlds, as one that has pointed out the shortest path between those already known.

3.—The practice of Great-Circle Sailing *must become general.* The "Go-a-head" propensity of the age will demand its adoption. The calls of even Class Interests will give an impetus to the movement. The expences of the Ship-Owner will be reduced in proportion as the duration of a voyage is shortened. In the same ratio will the Merchant gain, by getting his wares earlier into the Market. The voyager will the sooner reach his port, and the Emigrant his future home; whilst the Mariner will himself experience increased satisfaction from the consciousness that his skill contributes not only to his own comfort, but to the well being of his fellow-men.

4.—That the arc of a Great Circle is the shortest distance between any two places on the surface of the globe, has long been known to most well-instructed, practical seamen; and that if a voyage could be conducted on the principles of Great-Circle Sailing, it would invariably be shorter than one pursued on a common rhumb. Yet it is a fact that hitherto the art has been but little practised, from the circumstance of the course being continually changing. The idea that it was absolutely necessary to find this constant change, has prevented its adoption as a necessary part of a Nautical Education. The working of the necessary rules has been deemed both difficult and tedious, and as the wind would not at all times allow of a Sailing Vessel being kept on this course, the Masters of these ships have very rarely had sufficient resolution to conquer the supposed difficulties attendant on the art. These difficulties have, however, been greatly lessened, voyages have been made on the principle, and the public has become aware that great advantages would accrue if this branch of Navigation should be brought into general practice. The Admiralty have to their honour published a set of useful Tables, calculated by Mr. J. Towson, of Devonport, by the aid of which a person may almost by inspection, prosecute a voyage approximate to a Great Circle. Yet these Tables do not meet the wants of our Merchant Marine. The calculations are made to vertices, and vertex-distances, varying by a whole

degree, consequently the latitudes corresponding to given longitudes, or given vertex-distances, may, unless corrected by interpolation, for which there are no directions, be given erroneously as much as 30 miles. From the same cause a considerable error will be found in the distance, but this is of less importance than errors in the latitudes. Nevertheless these Tables are of sterling value. They have paved the way for the adoption of more accurate modes of calculation, and may still be used as a check against material errors.

5.—To offer a remedy for these defects, and still make Great-Circle Sailing practical and easy, is the object of this little work. The rules given are simple, easy to be understood, and easy to be worked. They are based on the principles of pure Spherics, the results may therefore be relied on. No other Tables are required than those of the common Logarithmic Arcs, which are published in every Epitome of Navigation. As practicability is aimed at rather than Mathematical niceties, no demonstrations of the rules are given, because they would be unintelligible to the merely *practical* man; the mathematician is aware of their truth, to him therefore a demonstration is unnecessary. The logarithms in the Examples are only taken out to four places of Decimals, which will generally be sufficient to give the arcs correct to the nearest minute, and the distance to the nearest mile; this will be quite near enough for practical purposes, but if the arcs should be required to seconds, a greater number of decimals may be employed. The courses and distances between point and point, may be found by the method used in common Navigation.

6.—DEFINITIONS.

I.—A GREAT CIRCLE is the largest that can be drawn on the surface of a sphere. It divides that sphere into two *hemispheres*, and if it were cut through in the direction of a Great Circle, the plane would pass through the Centre.

II.—A Great Circle can be drawn through any *two points* placed at random on the surface of a sphere; the arc that unites them is the *shortest line* that can be drawn between them. Hence the arc of a Great Circle is the only direct course between two places on the globe.

III.—All GREAT CIRCLES drawn on the surface of a sphere bisect each other.

IV.—The MERIDIANS are Great Circles drawn on the surface of the Globe: they bisect each other at the poles.

V.—The EQUATOR is a Great Circle every where equidistant from the poles; all the meridians meet it at Right Angles.

VI.—The EQUATORIAL points are those in which any Great Circle cuts the Equator; they are 180° distant from each other, and 90° distant from the meridian passing through the points of their greatest separation.

VII.—The VERTEX is the point of greatest separation of a Great Circle from the Equator; its measure is the arc of the meridian passing through it, and is identical to its Latitude.

VIII.—The VERTEX-DISTANCE is an arc of the Equator lying between the meridian passing through the vertex, and any other meridian; it is their difference of Longitude.

IX.—An INTERMEDIATE POINT is one lying in a Great Circle, of which the Latitude and Longitude are either known or can be found.

X.—COURSE is the angle which a Tangent to a Great Circle makes with the meridian passing through the point of contact. This angle changes with each meridian cut by the circle, and has been the chief obstacle to the practice of Great-Circle Sailing.

7.—Rules and Examples for Simple Courses.

For the Courses.

Rule I.—Find half the Difference of Longitude, between the two places, and also the half sum and half difference of their Colatitudes. If the places are on opposite sides of the Equator 90°. must be added to the latitnde of one of them for its Colatitude.

II.—Add together the *Log. Secant* of the half sum of the Colatitudes, the *Log. Cosine* of half their difference, and the *Log. Cotangent* of half the difference of Longitude; the sum, rejecting tens from the index, will be the *Log. Tangent* of half the sum of the courses.

III.—Add together the *Log. Cosecant* of half the sum, the *Log. Sine* of half the difference of the Colatitudes, and the *Log. Cotangent* of half the difference of Longitudes; the sum rejecting tens from the index, will give the *Log. Tangent* of half the difference of the Courses. As the arcs are the same in both these formulæ the Logarithms of each arc may be taken out at the same opening of the Tables.

IV.—The sum of these two arcs will be the Course from the place which has the greater latitude, and their difference the course from the

other place. These courses are to be reckoned from the pole of the latitude of the place. They are to be considered merely as auxiliary arcs in succeeding operations, and not as indicating the direction in which it is required to sail.

8. *For the Distance.*

Rule.—Add together the *Log. Cosine* of half the sum of the Co urses found by the above Rule, the *Log. Secant* of half their difference, and the *Log. Tangent* of half the sum of the Colatitudes; the sum rejecting tens, will be the *Log. Tangent* of half the distance. This arc multiplied by 2, and reduced to minutes, will give the distance in miles. The Logarithms of this operation may be taken out at the same time as those of the same arcs in the preceding rules.

9. *For the Latitude of the Vertex.*

Rule.—To the *Log. Sine* of the Course, from the place nearest to the Equator, add the *Log. Cosine* of the latitude of that place, the sum, rejecting ten, will be the *Log. Cosine* of the Latitude of the Vertex.

10. *For the Longitude of the Vertex.*

Rule.—To the *Log. Tangent* of the above Course, add the *Log. Sine* of the Latitude of the place; the sum, rejecting ten, will be the *Log. Cotangent* of the Vertex-Distance. By applying this arc to the Longitude of the place, the Longitude of the Vertex will be known. The Equatorial Points, being 90°. distance from the Vertex, may now, if required, be ascertained, by applying to the Vertex-Distance, its complement.

11. *To find the Latitude of any number of Intermediate points.*

Rule.—To the *Log. Tangent* of the Vertex, add the *Log. Cosine* of the Vertex-Distance of any assumed Longitude, the sum, less ten, will be the *Log. Tangent* of the corresponding Latitude. The Log. Tangent of the Vertex will be a constant quantity in the computation of any number of Intermediate points.

12.—When the Latitude and Longitude of any Intermediate point is found, it may be laid down on a Mercator's Chart,* and if the arc of a circle be drawn through this point, and the Ports of departure and destination, the Curve will represent the Great-Circle Course between these

* The Centre of the arc passing through any three points, may be found by bisecting the two Chords perpendicularly; the points of intersection will be the Centre of the arc.

Ports. The course and distance from any one point in it to another, may be then found quite near enough for practice, by the help of a parallel ruler and a pair of compasses, in the same manner that the bearing and distance of any point of Land from the Ship, are usually found. It will, however, be a better plan to find a number of Intermediate points by calculation, and place them either on a Chart, or in a Tabular form, as will hereafter be shown, and the Course and distance between each, may be found either by the Traverse Tables or Trigonometry.

13—Ex. A ship is off Lundy Island, in lat. 51° 10 'N. long. 4° 38′ W. and is bound to Jamaica; required the Course, Distance, Latitude and Longitude of the Vertex, Equatorial Points, and Intermediate Points for every tenth degree of longitude, in the Great Circle, passing through the ship's place and Morant Point in Latitude 58° 17′ N. and Longitude 76° 5′ W.

	deg. min.	*deg min*		*deg. min.*		*deg. min.*
Lundy Island	51 10 N.	4 38 W.	Colatitudes	38 50		
Morant Point	17 58 N.	76 5 W:		72 2		
	Diff. of Long.	71 27	Sum	110 52	Half	55 26
	Half do.	35 43½	Diff.	32 12	Do.	16 36

For the Courses	*deg. min.*					For the Distance	*deg. min.*		
Half Sum of Colats	55 26	Sec.	0.2461	Cosec	0.0844	½ sum of courses	66 65	Cos.	9.5931
Half diff. do.	16 36	Cos.	9.9815	Sin.	9.4559	½ diff. do.	25 45	Sec.	0.0454
Half diff. Long.	35 43	Cot.	0.1432	Cot.	0·1432	½ sum of colats	55 26	Tan.	0 1618
Half sum of Courses	66 56	Tan.	10.3708			Half distance	32 16	Tan,	9.8003
Half diff. do.	25 45	...	...	Tan.	9.6835		2		
Course from Lundy N. 92 41 W. or S. 87 19 W.						Distance	64 32=3872 miles		
Do. Morant Pt. N. 41 11 E.									

For the Lat. of Vertex	*deg. min.*			For the Long. of Vertex.	*deg. min.*		
Course from Morant Point	41 11	Sin.	9.8185	... ...	...	Tan.	9.9420
Latitude of do.	17 58	Cos.	9.9783	... ...	...	Sin.	9.4892
Latitude of Vertex	51 13	Cos.	9.7968	Vertex Distance	74 54	Cot.	9.4312
				Long. of Mor. Pt.	76 5		
				Long, of Vertex	1 11 W.		

Equatorial Points 91 deg. 11 min. W. and 88 deg. 49 min. E.

For the Intermediate Points.

		deg min		*deg min*		*deg min*		*deg min*		*deg min*	
Long W		10 0		20 0		30 0		40 0		50 0	
Ver. dis	Cos.	8 49	9.9948	18 49	9.9761	28 49	9.9426	38 49	9.8916	48 49	9.8185
Vertex	Tan.	51 13	0.0950	... ...	0.0950	... ...	0.0950	... ...	0.0950	... ...	0.0950
Lat. N.	Tan.	50 53	10.0898	49 40	10.0711	47 29	10.0376	44 7	9.9866	39 20	9.9135

		deg min		*deg min*	
Longitudes		60 0		70 0	
Vertex dist.	Cos.	58 49	9.7114	68 49	9.5579
Vertex	Tan.	51 13	0.0950	... ...	0.0950
Latitudes N	Tan.	32 28	9.8064	24 13	9.6529

By Mercator the course from Lundy is S. 59 deg. 51 min. W. and distance 3967 miles. Diff. in favour of G. C. Course 95 miles.

14.—Ex. A ship is off the Land's End, in Lat. 50° N. Long. 5° 40′ W. It is required to find the elements of the Great-Circle Arc between the place of the ship and Cape Race, Newfoundland in Lat. 46° 40′ N. Long. 53° W.

	deg min	*deg min*		*deg min*	*deg. min.*
Land's End	50 0 N.	5 40 W.	Colatitudes	40 0	
Cape Race	46 40 N.	53 0 W.		43 20	
Diff. of Long.		47 20	Sum	83 20	Half 41 40
Half ditto		23 40	Diff.	3 20	Do. 1 40

For the Courses

	deg min				
Half Sum of Colats	41 40	Sec.	0.1267	Cosec	0.1773
Half diff. do.	1 40	Cos.	9.9998	Sin.	8.4637
Half diff. Long	23 40	Cot.	0.3583	Cot.	0.3583
Half Sum of Courses	71 52	Tan.	10.4848		
Half diff. do.	5 42	...	...	Tan.	8.9993
Cour. from L. End	N. 77 34 W.				
Do. Cape Race	N. 66 10 E.				

For the Distance

	deg min		
½ sum of courses	71 53	Cos.	9.4931
½ diff do.	5 42	Sec.	0.0022
½ sum of colats	41 40	Tan.	9.9494
Half Distance	15 33½	Tan.	9.4447
	2		
Distance	31 7=1867 miles.		

For the Lat. of Vertex

	deg min		
Course from Cape Race	66 10	Sin.	9.9613
Latitude of do.	46 40	Cos.	9.8365
Latitude of Vertex	51 7	Cos.	9.7978

For the Long of Vertex

	deg min		
... ...		Tan.	0.3548
... ...		Sin.	9.8618
Vertex distance	31 16	Cot.	10.2166
Long of C. Race	53 0		
Long of Vertex	21 44 W.		

The Equatorial Points are 111 deg. 44 min. W. and 68 deg. 16 min. E.

For the Intermediate Points.

		deg min		*deg min*		*deg min*		*deg min*		*deg min*	
Lon. W.		10 0		15 0		20 0		25 0		30 0	
Ver. dis.	Cos.	11 4	9.9908	6 44	9.9970	1 44	9.9998	3 16	9.9993	8 16	9.9955
Vertex	Tan.	51 7	0.0934	... ...	0.0934	... ...	0.0934	... ...	0.0934	... ...	0.0934
Lat. N.	Tan.	50 31	10.0842	50 55	10.0904	51 6	10.0932	51 4	10.0927	50 50	10.0889

		deg min		*deg min*		*deg min*		*deg min*	
Longitudes W.		35 0		40 0		45 0		50 0	
Vertex dist.	Cos.	13 16	9.9883	18 16	9.9775	23 16	9.9632	28 16	9.9449
Vertex	Tan.	51 7	0.0934	... ...	0.0934	... ...	0.0934	... ...	0.0934
Latitudes N.	Tan.	50 22	10.0817	49 39	10.0709	48 43	10.0566	47 31	10.0383

Course by Mercator is S. 83 deg. 5 min. W.: Distance 1889 miles. In this example the Vertex being between the two ports, the Vertex-distances first decrease till the Long. of the Vertex is attained, after which they increase.

15.—Ex. Required the Elements of the Great-Circle Course passing through the point off Cape Spartel in Lat. 35 deg. 56 min. N., Long. 5 deg. 50 min. W., and Halifax in Lat. 44 deg. 44 N., Long. 63 deg. 36 min. W., for every fifth degree of Longitude.

	deg. min.		deg. min.		deg. min.		deg. min.
Cape Spartel	35 56 N.		5 50 W.	Colats	54 4		
Halifax	44 44 N.		63 36 W.		45 16		
		Diff. Long.	57 46	Sum	99 20	Half	49 40
		Half Do.	28 53	Diff.	8 48	Do.	4 24

For the Courses.

	deg min		
Half Sum of Colats	49 40	Sec. 0.1889	Cosec 0.1179
Half Diff. do.	4 24	Cos. 9.9987	Sin. 8.8850
Half Diff. Long.	28 53	Cot. 0.2583	Cot. 0.2583
Half Sum	70 18	Tan. 10.4459	9.2612
Half Diff.	10 21		

Courses { N. 80 39 E. from Halifax
N. 59 56 W. from C. Spartel

For the Distance.

	deg min	
½ Sum	70 18	Cos. 9.5278
½ Diff.	10 21	Sec. 0.0071
½ Sum Colats	49 40	Tan. 0.0711
Half Dist.	21 59	Tan. 9.6060
	2	

Distance 43 58 = 2,638 miles.

For the Lat. of Vertex. For the Vertex Dist.

	deg min			deg min	
Course from C. Spartel	59 56	Sin. 9.9373	... ...	...	Tan. 0.2374
Latitude of do.	35 56	Cos. 9.9083	... ...	...	Sin. 9.7685
Latitude of Vertex	45 30	Cos. 9.8456	Vertex Distance	44 37	Cot. 10.0059
			Long. of C. Spartel	5 50	
			Long. of Vertex	50 27 W.	

Equatorial Points 39 deg. 33 min. E., and 104 deg. 27 min. W.

For the Intermediate Points.

	deg. min.	deg. min.	deg. min.	deg. min.	deg. min.
Long. W.	5 50	10 0	15 0	20 0	25 0
Ver. Dist.	Cos. 44 37 9.8524	40 27 9.8814	35 27 9.9101	30 27 9.9355	25 27 9.9557
Vertex	Tan. 45 30 0.0076	... ... 0.0076	... ... 0.0076	... ... 0.0076	... ... 0.0076
Latitudes	Tan. 35 56 9.8600	37 45 9.8890	39 36 9.9177	41 16 9.9431	42 35 9.9633

	deg. min.	deg. min.	deg. min.	deg. min.	deg. min.
Long W.	30 0	35 0	40 0	45 0	50 0
Ver. dist.	Cos. 20 27 9.9717	15 27 9.9840	10 27 9.9927	5 27 9.9980	0 27 9.9999
Vertex	Tan. 45 30 0.0076	... ... 0.0076	... ... 0.0076	... ... 0.0076	... ... 0.0076
Lats.	Tan. 43 38 9.9793	44 27 9.9916	45 1 10.0003	45 22 10.0056	45 30 10.0075

		deg. min.		deg. min.		deg. min.	
Long. W.		55 0		60 0		63 36	
Vertex Dist.	Cos.	4 33	9.9986	9 33	9.9939	13 19	9.9882
Vertex	Tan.	45 30	0.0076	... ...	0.0076	... ...	0.0076
Latitudes N.	Tan.	45 25	10.0062	45 6	10.0015	44 44	9.9958

16.—Ex. Required the elements of the Great-Circle Course from Panama Lat. 8 deg. 57 min. N. Long. 79 deg. W. and Port Jackson Lat. 33 deg. 51 min. S. Long. 151 deg. E.

	deg min		deg min		deg min		deg min
Panama	8 57 N.		79 0 W.	Colats	98 57		
Port Jackson	33 51 S.		151 0 E.		56 9		
		Diff. of Long	230 0		155 6	Half	77 33
		or	130 0		42 48	Do:	21 24
		Half ditto	65 0				

By working this Example in the same manner as the foregoing ones, the half sum of the courses will be 63 deg. 36 min., and the half diff. 9 deg 53 min.; S. 73 deg. 29 min. E. the the course from Port Jackson, and S. 53 deg. 43 min. W. the course from Panama. The latitude of the Vertex is 37 deg. 13 min., and its longitude 58 min. W. or 179 deg. 2 min. E. The Intermediate Points, and their Vertex-Distances for every tenth degree of longitude are shewn in the following Table.

	deg min	deg min	deg min	deg min	deg min	deg min	deg min	deg min
Longitudes W	79 0	80 0	90 0	100 0	110 0	120 0	130 0	140 0
Vertex dist.	76 2	79 2	89 2	80 58	70 58	60 58	50 58	40 58
Latitudes N.	8 57	8 13	0 44	6 48S	13 15	20 14	25 34	29 50

	deg min	deg min	deg min	deg min	deg min	deg min	deg min
Longitudes W.	150 0	160 0	170 0	180 0	170 0 E.	160 0	151 0
Vertex dist.	30 58	20 58	10 58	0 58	9 2	19 2	28 2
Latitudes S.	33 4	35 21	36 42	37 13	36 52	35 41	33 50

17.—Compound Courses. It may often occur that the simple Great-Circle course between two ports will be impracticable, in consequence of the interposition of Islands, Headlands, and other obstacles. Under such circumstances it would be advisable to divide the voyage into two or more parts, directing the Great-Circle courses so as to clear each impediment in succession. The following examples will amply illustrate this kind of sailing.

18.—Ex. Required the Great-Circle Courses from the Land's End to Block Island, in latitude 41° 10′ North, and longitude 71° 39′ West.

As the direct Great-Circle Course in this voyage would lead over the Southern coast of Newfoundland, it is impracticable, it will therefore be best to adopt the course to Cape Race as shown in Ex. par. (14.) The arc of a second great circle passing through this point and B. Island may next be laid down, of which, the intermediate points for the longitudes 55°, 60°, 65° and 70° West, are 46° 16′, 45° 4′, 43° 36′, and 41° 49′ North. The distance on this arc is 14° 28′=868 miles, the distance on the first arc is 1867 miles. Total 2735 miles.

Should this last arc be supposed to approach too near the coast of Nova Scotia, the course may be directed to any other series of latitudes lying between the common rhumb and this arc, but in doing so the distance will be increased proportionately with the deviation.

19.—When the points of departure and destination are on opposite sides of the Equator, and the *direct* Great-Circle Course is impracticable, it would be best to fix upon some point in the Equator for crossing it, so that at the least, one of the arcs may be a simple one. If any impediment should exist in either course that would prevent its being carried direct to or from the Equatorial point, the best plan would be

to divide it so that the course from the port should be directed so as just to clear the impediment, and meet the meridian passing through the Equatorial point as near the Equator as possible, when the rest may be run on that meridian.

20.—When a Great-Circle Course leads to or from a known point of the Equator, the following mode of finding the Courses, Distance, and Vertex, will be more easy than by the rules given in (7) (8) and (9).

Rule 1.—Find the difference of longitude between the Equatorial point and the port, then set down separately the *log. sine,* the *log. cosine* and the *log. cotangent* of this difference of longitude.

2.—Under these three Logarithms write respectively the *log. cotangent,* the *log. cosine,* and the *log. sine* of the latitude of the port.

3. Add together each of these pairs of Logarithms, rejecting ten from the index of each; the first sum will be the *log. cotangent* of the latitude of the Vertex, the second the *log. cosine* of the distance, and the third the *log. cotangent* of the course from the Port; to be reckoned from the opposite pole, if the difference of longitude is less than 90°; but from its own pole if greater. The course from the Equator is the complement of the latitude of the Vertex. The longitude of the Vertex will be 90° distant from the Equatorial Point. In finding the Distance by this Rule, if the difference of Long. is greater than 90° the supplement of the arc found is to be taken.

21.—Example. Required the particulars of the Great-Circle Courses from the Land's End to Cape Horn, in latitude 55° 55′ S. and longitude 67° 21′ W. The first course to be directed to latitude 17° N. and longitude 25° W. The Equator to be crossed in 25° W.

Remark. The direct course to Cape Horn would lead across the eastern coast of Brazil and is therefore impracticable. The direct course from the Land's End to the Equator in 25° W. would lead to the eastward of the Cape Verd Islands, a part of the Atlantic to be avoided by reason of the peculiarities of its meteorology; on this account the first course has been directed to Lat. 17° N. It is not presumed that the Courses laid down either in this or the following examples are the best that could be given: to plan any voyage is more the province of the practical mariner than the mathematician.

	deg min	*deg min*		*deg*		*deg min*
Land's End	50 0 N.	5 40 W.	Colats {	40		
First Point of Destination	17 0 N.	25 0 W.		73		
Difference of Long. ...	... ...	19 20	Sum	113	Half	56 30
Half Do. ...	... ...	9 40	Diff.	33	Do.	16 30

For the Courses	deg min					For the Distance	deg min	
Half sum of Colats	56 30	Sec.	0.2581	Cosec	0.0789	½ sum of courses	84 24	Cos. 8.9894
Half diff. do.	16 30	Cos.	9.9817	Sin.	9.4533	½ diff. do.	63 26	Sec. 0.3495
Half diff. Long.	9 40	Tan.	0.7687	Cot.	0.7687	½ sum of col.	56 30	Tan. 0.1792
Half sum of Cours.	84 24	Tan.	11.0085			Half Dist.	18 15	Tan. 9.5181
Half diff do.	63 26			Tan.	10.3009		2	
Land's End	N. 147 50 W. or S. 32 deg. 10 min. W.					Distance	36 30=2190 miles	
1st. Point	N. 20 58 E							

For the Vertex.	deg min					
Course from 1st. Point	20 58	Sin. 9.5537	... ...			Tan. 9.5834
Latitude of do.	17 0	Cos. 9.9806	... ...			Sin. 9.4659
				deg min		
Latitude of Vertex	69 59	Cos. 9.5343	Vertex Distance	83 36		Cot. 9.0493
			Long. of 1st. Pt.	25 0 W.		
			Long of Vertex	58 36 E.		

For the Intermediate Points

		deg	min		deg	min		deg	min		deg	min	
Longitudes W.		10	0		15	0		20	0		25	0	
Vertex Dist.	Cos.	68	36	9.5621	73	36	9.4508	78	36	9.2959	83	36	9.0472
Lat. of Vertex	Tan.	69	59	0.4385	...	...	0.4385	...	...	0.4385	...	...	0.4385
Latitudes N.	Tan.	45	4	10.0006	37	47	9.8893	28	29	9.7344	17	0	9.4857

The Distance from 1*st. Point, to the Equator, being on a Meridian, a Great Circle, is* 17 *deg.*=1020 *miles.*

From the Equator to Cape Horn.

	deg min		deg min
Equator	0 0		25 0 W.
Cape Horn	55 55 S.		67 21 W.
		Diff. Long.	42 21

	For the Vertex deg min				For the Distance		Course from C. Horn
Diff. Long.	42 21	Sin. 9.8284	... ...		Cos. 9.8687	... ...	Cot. 0.0402
Lat. of Cape	55 55	Cot. 9.8303	... ...		Cos. 9.7485	... ...	Sin. 9.9181
			deg min			deg min	
Lat. of Vertex	65 30.	Cot. 9.6587	65 32		Cos. 9.6172	N. 47 45 E.	Cot. 9.9583

Course from the Equator is S. 24 30 W., the complement of the Vertex.—Dist. 3932 miles.

The Intermediate Points are set in the following Table without the Logarithms.

	deg	min	deg	min	deg	min	deg	min	deg	min	deg	min	deg	min	deg	min
Long. W.	30	0	35	0	40	0	45	0	50	0	55	0	60	0	65	0
Vertex dist.	85	0	80	0	75	0	70	0	65	0	60	0	55	0	50	0
Latitude S.	10	50	20	52	29	36	36	54	42	50	47	39	51	32	54	40

Dist. 1st arc.	2190	miles
2nd arc.	1020	—
3rd arc.	3932	—
Total	7142	

22.—Composite Sailing. Is resorted to when the circumstances of a Voyage render it expedient for one part of it to be performed on a Common Rhumb. Thus, when the difference of the Longitude between two places is large, the Vertex of the Great Circle is in a Latitude too high to be practicable. In such a case it would be desirable to fix on a *Maximum Latitude*, above which the Ship is not to go. The Voyage

will then be divided into three parts, of which the first and third will be arcs of Great Circles; the Vertex of each being the Maximum Latitude, but having different Longitudes. The second or middle part will be run on the Parallel of the Maximum Latitude for a distance equal to the difference of Longitude between the Vertices.

23—In this case the Latitude of the Vertex being known, its Longitude may be most readily found by the following Rule. To the *Log. Tangent* of the Latitude of the Port, add the *Log. Cotangent* of the Latitude of the Vertex, the sum, rejecting ten, will be the *Log. Cosine* of the Vertex distance of the Port, which if applied to the Longitude of the Port will give the Longitude of the Vertex. The distance also may be found from the same arcs, by adding to the *Log. Sine* of the Latitude of the Port, the *Log. Cosecant* of the Latitude of the Vertex; the sum, rejecting ten, will be the *Log. Cosine* of the distance of the port from the parallel of the maximum latitude. The Distance to be sailed on the parallel of the maximum latitude, may be found by the common rule for parallel sailing. Sufficient materials to illustrate this kind of sailing will be found in the following Example.

24.—Ex. Suppose a Ship off the Land's End is bound for King's Island, in lat. 39° 30′ S. Long. 143 50′ E. She proposes to cross the Equator in Long 25° W. and to fix her maximum latitude to 51° S. Required the particulars necessary to delineate the several courses of the voyage on a Mercator's Chart.

The first two courses of this voyage are worked in Ex. (21.) where the distance to the Equator is shown to be 3,210 miles.

The second course from the Equator to the parallel of the maximum latitude, will be the hypothenuse of a right-angled and quadrantal spherical triangle, of which the *latitude of the vertex* is 51°, the same as the maximum latitude, and its *longitude* 65° E. being 90° distant from 25° W. the *equatorial point*. The course from the Equator S. 39° E. the complement of the vertex, to 90°; and the distance is a quadrant or 90°= 5,400 miles. The intermediate points are to be found by rule (11); they are shown for every tenth degree of longitude in the following table.

Table of Intermediate Points

	deg min	deg min	deg min	deg min	deg min	deg min	deg min	deg min	deg min
Longitudes	20 W.	10	0	10 E.	20	30	40	50	60
Vertex dist.	85	75	65	55	45	35	25	15	5 0
Latitudes S	6 9	17 43	27 33	30 9	41 8	45 20	48 13	50 1	50 54

The particulars of the last course are now to be found.

	deg min			
Latitude of King's Island	39 30	Tan. 9.9161	... ...	Sin 9.8035
Do. Vertex	51 0	Cot. 9.9084	... ...	Cosec. 0.1065
			deg min	
Vertex Distance	48 7	Cos. 9.8245	Distance 35 4	Cos. 9.0190
Longitude of Port	143 50 E.		60	
			2104 miles.	
Long of 2nd Vertex	95 43 E.		For the Distance on the Parallel.	
Long of 1st Vertex	65 0 E.		1843 3.2655	
			Cos. 51 deg. 9.7989	
Diff. of Long on the parallel	30 43=1843 miles			
			Distance on Parallel 3.0644	1160 miles.
Distance from Land's End to Equator				3210 "
From Equator to Max. Latitude				5400 "
Distance from Parallel to Destination				2104 "
Total Distance				11874

Table of Intermediate Points.

	deg min	deg min	deg min	deg min	deg min
Longitudes E.	100 43	110 43	120 43	130 43	140 43
Vertex dist.	5 0	15 0	25 0	35 0	45 0
Latitudes S.	50 54	50 1	48 13	45 20	41 8

In making a Table of Intermediate Points for any particular voyage, the Vertex-Distance need not be inserted as the latitudes and longitudes are all that can be required either to mark the point on a Chart or to find the course.

25.—In the preceding examples, specimens have been given of all that can be required in *practical* Great-Circle Sailing; and whether it is proposed to sketch the courses on the Chart, by finding only one intermediate point, as proposed in par. (12.), or by laying down the positions of a greater number of points found by calculation; the practice, will after a short study become easy and expeditious. The Tyro may rely on the accuracy of the last plan,as the latitudes of the vertex and intermediate points may be obtained to the nearest *minute* quite as easily as to the nearest to degree. This is of the greatest importance, for should there be an error in the vertex, there will be a corresponding error not only in the distance, but also in the latitude corresponding to any degree of longitude; if the vertices are only calculated to whole degrees, this error may be as much as *thirty miles*.

26.—Independently of shortening the *distance* of a voyage by following the Great-Circle course as closely as possible, a great advantage will be gained by giving to the mariner a *fixedness of purpose* in the prosecution of his voyage, which may prevent him from going considerably *out of his way*. His plan has been laid, and well considered before he sets sail. To the whisperings of caprice he will give no ear, nor will he be easily diverted by any ignis fatuus purpose that would entice him to pursue a course contrary to his first design, in the hope of

obtaining more favourable winds, from the consciousness that such a deviation would certainly lengthen the distance, whilst the compensating favouring breeze might be denied. There do, indeed, exist several peculiarities of winds in various tracts of the Ocean, which it may be prudent either to avoid or take advantage of; but the wise and well instructed mariner will well weigh the certainty of the advantage before he commences his voyage. In the Example (21,) the Great-Circle Arc has been thus diverted from its simple course to the Equator, and a compound one substituted, in which the greatest westing has been attained in latitude 17 N. Many mariners make it much earlier. Suppose it to be attained in lat. 35, by no means an uncommon thing, the distance sailed would then be 1,237 miles to the 25 degree, thence to the 17° N. 1,080 miles, total 2,317 miles or 127 miles more than by the Great Circle. Whereas, had the course by mercator been followed, the distance would not have exceeded the Great-Circle Course by three miles. Here the distance has been lengthened 6 miles in every 100, and not the slightest advantage gained to counterbalance the sacrifice of time and distance, a sacrifice which would have been avoided by a person who thoroughly appreciated the advantage of following up a well considered purpose. This naturally leads to the subject of the following paragraphs.

27.—Windward Sailing. This sailing is resorted to when the wind is in any degree adverse to the ship's course. This contingency forms one of the objections which are often made to the practice of Great-Circle Sailing; but if the subject be fairly considered, it presents one of the strongest proofs of the importance of a knowledge of the art. It often happens, that the course by the Rhumb deviates as much as three or four points from the true, or Great-Circle Course. Let us suppose a case, in which the course given by the Rhumb is W. S. W. The Great-Circle Course, W. by N. and the wind W. by S. A person unacquainted with the Great Circle, would natnrally choose the starboard-tack, as he would suppose that this course is only 5 points from the direction of the port, whilst in reality it is 8 points from it. Had he chosen the port-tack, which seems to take him 7 points from the direction of his port, he would in reality be proceeding only 4 points distant from the true course, and, consequently making a considerable approach to it, in the former case he makes none.

28.—The objection on account of an adverse wind, applies equally to the course given by the Rhumb, and to that given for the Great Circle;

so that this objection to the *practice of Great-Circle Sailing,* is groundless. But it may be doubted whether the rigid adherence to the true course in spite of opposing winds, is at all times advantageous. To decide this question, is perhaps more the province of the practical mariner, than the mathematician, but even in this case, calculation may throw some light on the subject, and its decision should be allowed its proper influence, till actual experiment shall have decided which is best. Let us suppose a case, the most unfavourable for pursuing a certain course. Two Ships are on the Equator, in 25° W. longitude, their destination Australia, the wind S. E. One of them proposes to follow the Great Circle tract pointed out in par. (24,) the other to proceed on the S. S. W. tack, until out of the influence of the opposing "Trade wind." Both ships sail at the rate of six knots per hour, consequently, in ten days they will each have sailed 1,440 miles. The Commander of the first ship will manage his traverse, so as to sail 755 miles on the S. S. W. rhumb, and 685 on the E. N. E. which will place him nearly on the Great Circle, in latitude 7°. 16′ S. and longitude 19°. 15′ W. He has made good 562 miles of his course, and is distant 1,629 miles from a point in the course on the meridian of Greenwich, in latitude 27° 33 S. a point which will probably be made by both ships. The other ship is supposed to have made no change in her course, and is in latitude 22°. 10′ S. longitude 34°.25.′ W. she has neared the above point only 298 miles, and is 1,895 miles distant from it or 266 miles more than the first ship. But which of them is in the best position, as it regards the rest of the voyage, especially if the second ship should proceed to the parallel of the maximum latitude on a Great Circle, cannot in consequence of the uncertainty of the wind, be mathematically demonstrated. The second ship is nearly out of the influence of the opposing trade wind, and variable winds may be expected; these may enable her to proceed direct towards her destination, but they may also prove adverse. The other ship will for some time longer have to struggle with the tradewind, but even that is not constantly fixed to one point; and as it is now directly opposed to her course any change must be in her favour. She steadily pursues her path, at a rate that chance cannot retard, but may accelerate.

29.—But trade winds do not pervade every part of the Ocean, and whether a ship proceed on a common rhumb or on a Great Circle, she has no reason to suppose the wind would for many days continue to blow from the same adverse point. It must after a time change, and it will

be a matter of prudent policy in the mariner, so to manage his course, that the expected change *must be in his favour*. This can only be done by keeping the point of his destination as much as possible in the direction of the wind, in Nautical phraseology, in "*The Wind's Eye*." This is a case for which no directions are given in books of common Navigation; but as the rigid adherence *to one course* is in this work strenuously advocated, it may be necesary to give the following rules, to enable the mariner to carry it into execution; they will be as applicable to sailing on a common rhumb as on a circle.

30.—To place the point of destination in the direction of the wind.

Rule 1.—After having found the bearing and distance of the point of destination, by the method practised in common Navigation; add together the *Log. Cosecant* of 6 points, the *Log. Sine*, of the angle, the wind makes with the bearing of the point of destination, and the *Log. of the Distance* of that point from the ship; the sum of these three Logarithms, rejecting tens, will be the *Log. of the distance* in miles, to be sailed to bring the wind to the same bearing with the point of destination.

2.—To find the whole distance.—To the *Log. Cosecant* of 4 points, add the *Log. Sine*, of the difference of the angle the wind makes with point of destination and 6 points, and the *Log. of the distance* of the ship from that point; the sum, rejecting the tens, will be the Log. of a number to be added to the distance before found. The sum will be the number of miles to be sailed, before she comes on the other tack, on which, if she sail a distance equal to the number of miles found by the second operation, she will have arrived at the point of destination. The sum of the miles sailed on both tacks will give the whole distance necessary to reach that point.

31.—To illustrate these rules, let us suppose a ship near Lundy Island, bound to Jamaica, the wind W. S. W. The first point of destination we will fix on in Lat. 50° 53′ N. Long. 10° W. The bearing of this point from the ship, is by Mercator Sailing, S. 85°. 12′ W. but that the work may be in points, we will say W. ½ S. or S 7½ W. points, and distance 203 miles. The angle this makes with the wind, is 1½ points, and the difference between this angle and 6 points, is 4½ points. Then by the above rules.

	6 pts. Cosec.	0.0344		4 pts. Cosec.	0.1505
	1½ pts. Sin.	9.4628		4½ pts. Sin.	9.8882
	203	2.3075		203	2.3075
1st. Dist.	64 miles.	1.8047	2nd. Dist.	222	2.3462
			1st. Dist.	64	
			3rd. Dist.	286 miles.	

Hence by sailing N. W. 64 miles the bearing of the point of destination from the Ship will be W. S. W. the same as the wind: nothing will be gained by going on the other tack till this is effected.

32—Having obtained these results the mariner will have it in his power to decide how he will manage his distances on each tack. He may either proportion his *time* in the ratio of the third and second distances, that is, he may sail 13½ hours on the N. W. course, and 10½ hours on the South; or he may, after sailing the first distance, 64 miles, run an equal number of miles on each tack. The former will at the end of every 24 hours place him on the chord that leads to his point of destination, the latter on the rhumb on which this point is in the "wind's eye." If equal distances are run on each of these modes, by the first, he will be nearest to the Great Circle, by the second, he will keep in such a position that any change of wind *must* be in his favour; either will keep him steady to his true course, and by no other mode of shaping it, would he be *placed so near his port*. He can have no reasonable hope that he should have a more favourable wind in the one position than in the other, consequently the track now pointed out *must be the best*. Should the wind continue to blow from the same point for 3 days 13 hours, he will have arrived at his point of destination, when he will have to shape a new course to another, not very distant, point in the Great Circle. Either of these modes only proposes to tack once in 24 hours, but it might be advantageous to tack twice in that time, if the wind is directly adverse.

33.—It may perhaps be as well to advert to another practice to which mariners often resort. It is, sailing away "from the wind" that the ship may go faster through the water. But from this practice no advantage can be gained unless the speed is so much accelerated as to place him *nearer* his port by pursuing it, than he would be by keeping close-hauled. Now if he sail 7 points from the wind his speed must be nearly doubled, to enable him to keep on a par with another ship sailing at 6 points.

For example. Suppose a ship when close-hauled to lie 6 points from the wind, the following table will shew the respective distances it must sail to arrive at a point of destination 100 miles distant, for every integer point of the angle which the course makes with the bearing of that point. Under these are placed the corresponding distances that must be passed over to arrive at the above point if it sail free at 6½ and 7 points from the wind.

TABLE.

Angle in points the course makes with the port.

Points from the wind.	0	1	2	3	4	5	6
6	100.	145.2	184.8	217.3	247.8	256.3	261.4
6½	132.8	191.4	243.6	286.4	318.3	337.9	344.5
7	196.2	284.8	362.5	426.4	473.6	502.7	512.6

Hence it appears that for every ten miles a ship sails when close-hauled, if it sail free half a point it must sail thirteen miles and a quarter, and if it go free a whole point, it will leave to sail 19 miles and a half, or at the respective rates of 5, 6½ and 9¾ knots per hour. This fact must convince every unprejudiced man, that in the case of adverse winds the most advantageous plan will be *to follow the true course, sailing as close to the wind as possible*. For the nearer a ship sails to the wind *the more rapidly she approaches her port*, though she has less way through the water. Thus it is evident that though the Mathematician has no power over the wind, yet he can point out the way to make even its inconstancy subservient to man's advantage.

34.—From the *constant* change of course, the rigid prosecution of a voyage on a Great Circle is, if not impossible, impracticable, but the approximation here recommended gives every advantage, the knowledge of its being the shortest distance between two ports can impart. The course here proposed is the *chord* connecting two not very distant points in the circle, of which the latitudes and longitudes have been ascertained by the preceding rules. If this chord be taken to arcs not greater than five degrees of longitude, the distance will not exceed the length of the arc itself, more than one tenth of a mile in every hundred. The bearing and distance of these two points are to be found by the ordinary modes, made use of to find the course and distance to the port of destination, in performing a "day's work" therefore any one competent to do this, can pursue the course pointed out.

35.—In definition X. the course found by the rules relating to the the Great Circle is described as being a tangent to the circle; this course, as stated in the preceding paragraph, is not to be followed. The chord course has these advantages over it, it is more easy to find, and when found it is certain to meet the Circle in the point proposed; whereas the Tangent Course takes the ship *out of the circle*, and each succeeding course is the tangent to a new one, having a different Vertex to the original circle, and to which it can only return on making the port. In the example par. (13) the chord course [illegible] the first point of destination is S. 85° 12′ W., and the distance 203 [illegible] the tan-

gent course is S. 87° 19′ W., and if the same distance [illegible] on this course, the ship would still be 8.5 miles from that point. Practically, this is a trifle in a run of 200 miles, but there can be nothing reasonable in wilfully or carelessly incurring this loss. In a run of 2000 miles the loss would amount to 85 miles. In an adverse wind it would take two days' sail to make it up.

If these tangent courses are changed only *by quarter points* they will give a track widely different from the original one; but the chord course by being carried a little beyond the Great Circle can easily be made to accommodate itself to this rate of change. The course found being the *true* one, must be reduced to the *magnetic* by applying the variation.

36.—Our plan is now laid before the public. Of its practicability there can be no doubt. The Great-Circle course is the shortest possible, and it often differs widely from the common rhumb. Through not knowing this, ships are frequently taken a circuitous route when one nearly direct might have been followed. When the wind is favourable there can exist no just reason why this true course should be departed from, and when it is adverse a judicious traverse not only makes the greatest progress, but places the ship in a position the most favourable to take advantage of any change of wind. Hence it is clear that by following this plan the duration of a voyage must be shortened. This is a thing of such importance to Ship Owners, Merchants, &c., that these gentlemen will be induced to *inform themselves* on this point, and if what is here proposed be found practicable, their *interest will induce them to promote its adoption.*

J. Wright, Printer, Steam Press, 18, Thomas Street, Bristol.